Astronomy

Kepler's laws

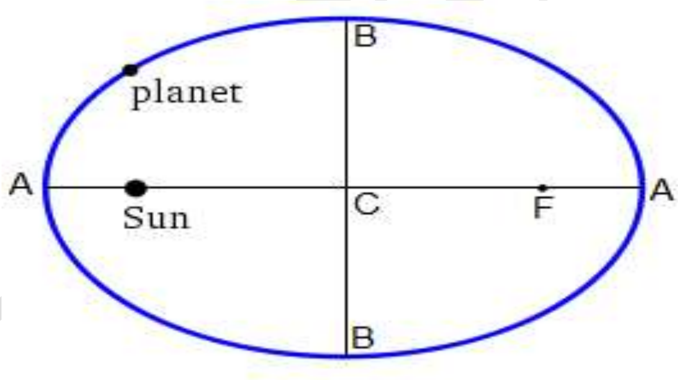
Kepler's first law: - The orbit of each planet is an ellipse with the Sun at one focus.

The following figure shows an elliptical planetary orbit with the sun at one of the foci. Then the line AA' is the major axis of the ellipse, C is the centre and therefore CA and CA' are the semi-major axes. Likewise BB' is the minor axis with CB and CB' are the semi-minor axes. If a and b denote the length of semi-major and semi-minor respectively then

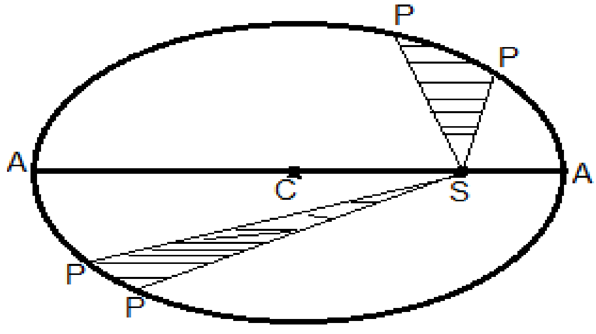
$$b^2 = a^2(1 - e^2) \ldots\ldots\ldots\ldots\ldots 1.1$$

Where e is the eccentricity of the ellipse, a quantity defined by the relation:

$$e = \frac{CS}{CA} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.2$$



Kepler's second law: - the Imaginary line joining the centre of a planet and the Sun sweeps out equal areas in equal intervals of time, i.e. the swept area per second is constant, as in figure below:



Kepler's third law: - the cubes of the semi-major axis of the planetary orbits are proportional to the square of the planets' period of revolution, i.e.

$$\boldsymbol{P^2 \propto a^3}$$

Where P is the period of revolution and a is the semi-major axis.

Newton's laws of motion

Newton's first law:- Every body continues in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an external impressed force.

Newton's second law: - the rate of change of momentum of the body is proportional to the impressed force and takes place in the direction in which the force acts i.e.

$$\frac{d(mv)}{dt} = F \dots\dots\dots\dots\dots\dots\dots\dots 1.3$$

Where F is the impressed force and v is the body velocity

Since $$a = \frac{dv}{dt} \quad \dots\dots\dots\dots\dots\dots\dots 1.4$$

So $$ma = F \quad \dots\dots\dots\dots\dots\dots\dots 1.5$$

Newton's third law: - To every action there is an equal and opposite reaction.

Newton's law of gravitation: - Every particle of matter in the universe attracts every other particle of matter with a force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

Hence for two particles separated by a distance r, the attraction force is given by

$$F = G\frac{m_1 m_2}{r^2} \quad \dots\dots\dots\dots\dots\dots\dots 1.6$$

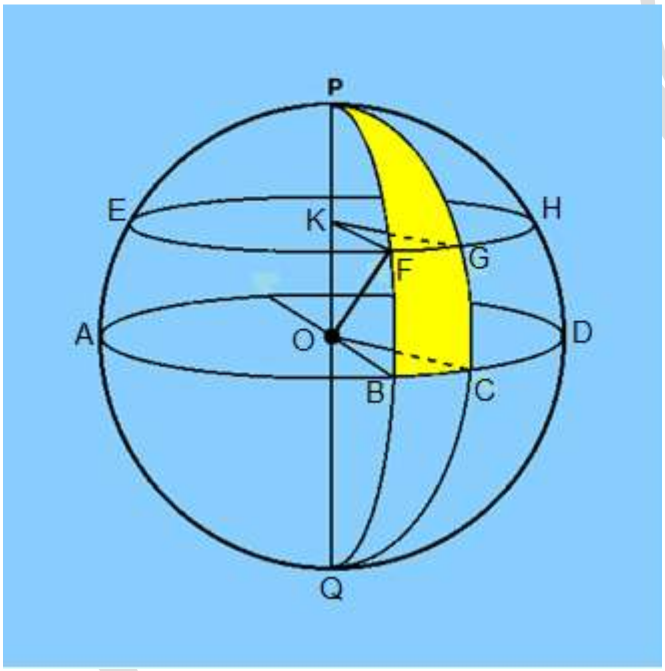
Where m_1, m_2 are the masses and G is the constant of proportionality.

$$G = 6.67 \times 10^{-11}\ \frac{N.m}{kg^2}$$

Spherical Geometry: - the geometry of sphere is made up of great circle, small circle and arcs of these figures.

Great circle: - is defined to be the intersection with the sphere of a plane containing the centre of the sphere. Since the centre is equidistant from all points on the sphere, the figure of the intersection must be a circle by definition.

Small circle: - is defined to be the intersection with the sphere of plane which is not containing the centre of the sphere.



In figure ABCDA is a great circle. If two points P and Q are chosen to be 90° away from all points on the great circle(by drowning the diameter POQ perpendicular to the plane ABCDA), they are said to be the poles of the great circle ABCDA.

The small circle EFGHE was obtained by chosen a point K on the diameter PQ and letting the plane through K and perpendicular to PQ cut the sphere. It is easily shown that the figure produced by this procedure is a circle.

Let PFBQP be any great circle through the poles P and Q. Then, by construction of EFGH, KF is perpendicular to OK, so that the triangle KFO is right-angled at K. By Pythagoras,

$$OF^2 = OK^2 + KF^2 \ldots\ldots\ldots\ldots\ldots\ldots.. 1.7$$

But OF and OK are both constant (OF is a radius of the sphere and OK is a constant) and, therefore, by equation (1.7), KF is constant.

Spherical angle: - the angle between the tangents at P to the great circle PFBQP and PGCQP. It is defined only with reference to two intersecting great circle.

If three great circles intersect one another so that a closed figure is formed by arcs of the great circle, it is called a spherical triangle provided that it possesses the following properties:

1- Any two sides are together greater than the third side.
2- The sum of the three angles is greater than 180°.
3- Each spherical angle is less than 180°.

In the figure above, figure PBC is an example of a spherical triangle. The side of the spherical triangle is expressed in angular measure. The length of the arc BC is given in terms of the angle θ it subtends at the centre of the sphere and the sphere's radius R by the relation

$$s = R \times \theta \ldots\ldots\ldots\ldots\ldots 1.8$$

Where θ is expressed in radians.

In the figure above, let r be the radius of the small circle EFGHE, so

$$FG = r \times < FKG° \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.9$$

Also $$BC = R \times < BOC \ldots\ldots\ldots\ldots\ldots 1.10$$

Both OB and KF lie on plane PFBQ, KF also lies in plane EFGH while OB lies on plane ABCD. Therefore, KF must be parallel to OB, since plane EFGH is parallel to plane ABCD. Similarly, KG parallel to OC. Then

$$< FKG = < BOC \ldots\ldots\ldots\ldots\ldots 1.11$$

Hence,

$$FG = BC \times \frac{r}{R} \ldots\ldots\ldots\ldots\ldots 1.12$$

In the plane triangle KOF, right-angled at K, KF=r; OF=R. Hence,

$$\mathrm{FG} = \mathrm{BC} \times \sin\mathrm{KOF} \ldots \ldots \ldots \ldots \ldots \ldots \ldots 1.13$$

But <POB=90° so that we may write alternatively

$$\mathrm{FG} = \mathrm{BC}\cos\mathrm{FB} \ldots \ldots \ldots \ldots \ldots .1.14$$

If the radius of the sphere is unity,

$$\mathrm{PF} =< \mathrm{POF} =< \mathrm{KOF} \ldots \ldots \ldots \ldots \ldots \ldots \ldots 1.15$$

and

$$\mathrm{FB} =< \mathrm{FOB} \ldots \ldots \ldots \ldots \ldots \ldots \ldots .1.16$$

so that we have

$$\mathrm{FG} = \mathrm{BC}\sin\mathrm{PF} \ldots \ldots \ldots \ldots \ldots ..1.17$$

and

$$\mathrm{FG} = \mathrm{BC}\cos\mathrm{FB} \ldots \ldots \ldots \ldots \ldots \ldots 1.18$$

Nautical mile:-

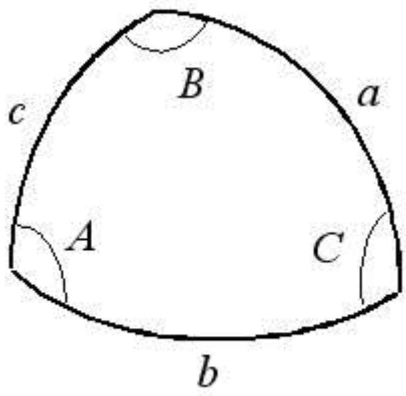
The nautical mile (nmi) is a unit of length that is about one minute of arc of latitude measured along any meridian, or about one minute of arc of longitude at the equator. It is 1,852 metres (approximately 6,076 feet).

The knot: - is a unit of speed equal to one nautical mile (1.852 km) per hour.

Spherical trigonometry:-

There are many formulas in the spherical trigonometry but four in particular are more often used than any others. They are the relations between the angles and sides of a spherical triangle and are invaluable in solving the problems that arise in spherical astronomy.

In the figure below ABC is a spherical triangle (A spherical triangle is a figure formed on the surface of a sphere by three great circular arcs intersecting pairwise in three vertices) with sides AB, BC, and CA of length c, a and b, respectively and with angles <CAB, <ABC and <BCA, hereafter referred to as angle A, B, and C respectively. The four formulas are:



1- **The cosine formula**:-

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \ldots\ldots\ldots\ldots 1.19$$

There are obviously two variations of this formula, namely

$$\cos b = \cos c \cos a + \sin c \sin a \cos B \ldots\ldots\ldots 1.19\text{ b}$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C \ldots\ldots\ldots\ldots 1.19\text{ c}$$

2- **The sine formula**:-

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 1.20$$

This formula must be used with care since, in being given a, b and B, for example, it is not possible to say whether A is acute or obtuse, unless other information is available, i.e. the formula gives A or (180-A).

3- **The analogue of the cosine formula**:-

$$\sin a \cos B = \cos b \sin c + \sin b \cos c \cos A \ldots\ldots\ldots\ldots\ldots 1.21$$

4- **The four-parts formula**:-

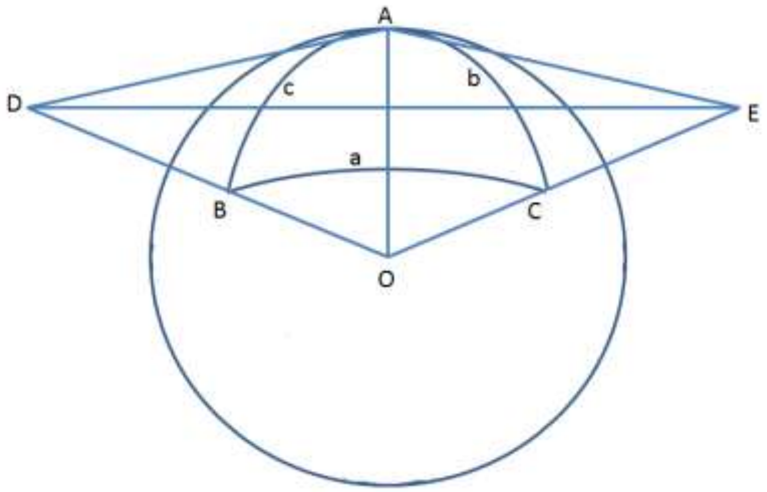
$$\cos a \cos C = \sin a \cot b - \sin C \cot B \ldots\ldots\ldots\ldots\ldots 1.22$$

Proof of cosine formula

In the figure below, let ABC be a spherical triangle with sides AB, BC and CA of lengths c, a and b respectively.

Draw tangents at A to the great circle arcs AB and AC to meet the radii OB and OC produced at D and E respectively. Join O and A, O being the centre of the sphere.

Then triangles ADE, ODE are plane triangles, though not in the same plane. By definition, <DAE is the spherical angle A. Also <DOE is the angle subtended by the arc BC. Hence, <DOE=a.



In ΔDAE we have

$\mathrm{DE}^2 = \mathrm{AD}^2 + \mathrm{AE}^2 - 2\mathrm{AD}.\mathrm{AE}\cos\mathrm{A} \ldots\ldots\ldots..1.23$

In ΔDOE we have

$$\mathrm{DE}^2 = \mathrm{OD}^2 + \mathrm{OE}^2 - 2\mathrm{OD}.\mathrm{OE}\cos\mathrm{a} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.1.24$$

Hence by subtraction

$$2\mathrm{OD}.\mathrm{OE}\cos\mathrm{a} = (\mathrm{OD}^2 - \mathrm{AD}^2) + (\mathrm{OE}^2 - \mathrm{AE}^2) + 2\mathrm{AD}.\mathrm{AE}\cos\mathrm{A} \ldots 1.25$$

Now ΔDAO is a plane angle right-angled at A, since DA is a tangent to the circle AB at A and OA is a radius of that circle. Hence, by Pythagoras,

$$OD^2 - AD^2 = AO^2 \ldots\ldots\ldots\ldots\ldots..1.26$$

Similarly, by considering ∆OAE, it is seen that

$$OE^2 - AE^2 = AO^2 \ldots\ldots\ldots\ldots\ldots\ldots..1.27$$

Hence, equation (1.25) becomes

$$OD.\,OE\cos a = AO^2 + AD.\,AE\cos A \ \ldots\ldots\ldots\ldots\ldots.1.28$$

Or

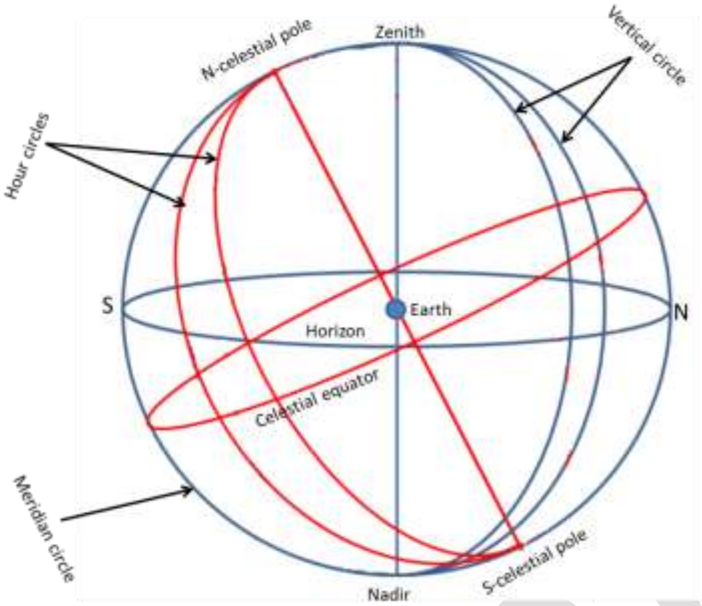
$$\cos a = \frac{OA}{OD}.\frac{OA}{OE} + \frac{AD}{OD}.\frac{AE}{OE}\ \cos A \ldots\ldots\ldots\ldots.1.29$$

In ∆DAO, OA/OD is the cosine of the angle <DOA while AD/OD is the sine of that angle. But <DOA=c, so OA/OD = cos c; AD/OD= sin c. Similarly, OA/OE= cos b; AE/OE= sin b.

Hence,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \ldots\ldots\ldots\ldots\ldots 1.30$$

Celestial sphere: - the celestial sphere is an imaginary sphere of arbitrarily large radius, concentric with the observer. All objects in the observer's sky can be thought of as projected upon the inside surface of the celestial sphere, as if it were the underside of a dome or a hemispherical screen. The celestial sphere is a practical tool for spherical astronomy, allowing observers to plot positions of objects in the sky when their distances are unknown or unimportant.



The zenith is an imaginary point directly above (in the vertical direction) a particular location, on the imaginary celestial sphere.

Nadir: - the point on the celestial sphere directly beneath a given position or observer and diametrically opposite the zenith.

Horizon: - is the great circle which is 90° away from the zenith and nadir.

Celestial poles: - The north and south celestial poles are the two imaginary points in the sky where the Earth's axis of rotation, indefinitely extended, intersects the celestial sphere. The north and south celestial poles appear permanently directly overhead to an observer at the Earth's North Pole and South Pole respectively.

Meridian circle: - is the great circle passing through the celestial poles and the zenith of a particular location.

Celestial equator: - - is a great circle on the imaginary celestial sphere, in the same plane as the Earth's equator. In other words, it is a projection of the terrestrial equator out into space.

Hour circle: - is the great circle through the object and the celestial poles. It is perpendicular to the celestial equator.

Vertical circle: - is a great circle on the celestial sphere that is perpendicular to the horizon. Therefore it passes through the zenith and the nadir.

Coordinate systems

The coordinate systems have a particular reference to great circles by which the direction of any celestial body can be defined uniquely at a given time. The choice of origin of the system also depends on the particular problem in hand. It may be the observer's position on the surface of the Earth (a topocentric system) or the Earth's centre (a geocentric system) or the Sun's centre (a heliocentric system) or, in the case of a certain satellite problem, the centre of a planet (a planetocentric system) or the centre of Moon (a selenocentric system).

1- **The horizontal (alt-azimuth) system**

This is the most primitive system, most immediately related to the observer's impression of being on a flat plane and at the centre of a vast hemisphere across which the heavenly bodies move.

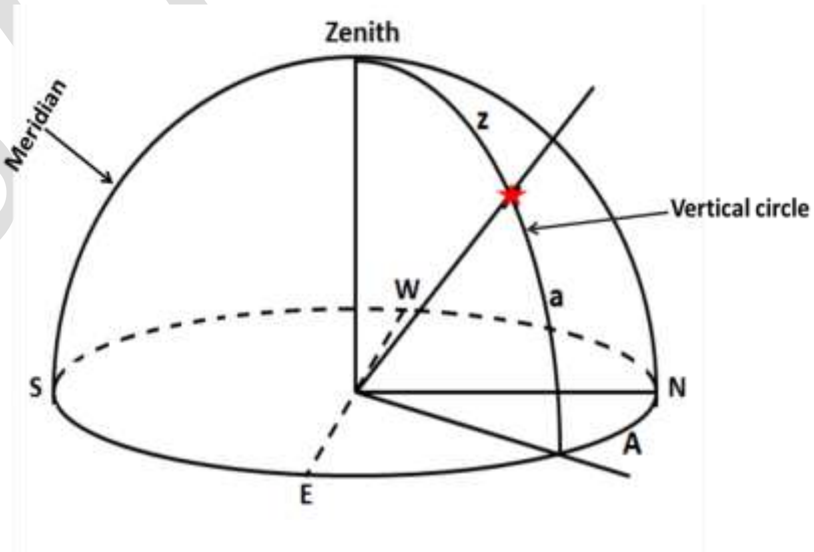
The two coordinates of this system are

a- **Altitude (a)**:- is the angular height of a celestial body from the horizon. It is measured in degree, arc-minute, arc-second. It is 0 at the horizon and 90° at zenith.

$$a = 90 - z \dots\dots\dots\dots\dots 1.46$$

Where z is the zenith distance which is the angular distance of the celestial object from the zenith.

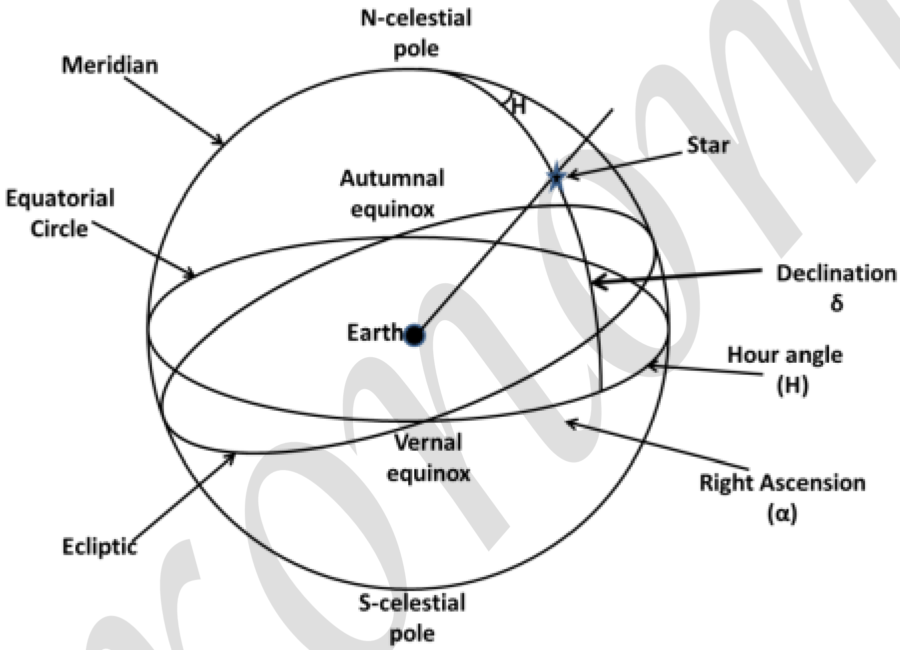
b- **Azimuth (A)**:- is the angle of the object around the horizon, usually measured from the north increasing towards the east. As in the figure below**:**



The mean disadvantage of this system of coordinates is that it is purely local. Two observers at different points on the Earth' surface will measure different altitudes and azimuths for the same object at same time. In addition, an observer will find the object's coordinates changing with time as the celestial sphere appears to rotate.

2- The equatorial system

If we extend the plane of the Earth's equator, it will cut the celestial sphere in a great circle called the celestial equator. It is the essential circle of the system. The essential coordinates are:-



a- The declination (δ): is the angular distance in degrees of the star from the equator along the meridian through the star. It is measured north and south of the equator from 0° to 90°, being taken to be positive when north.

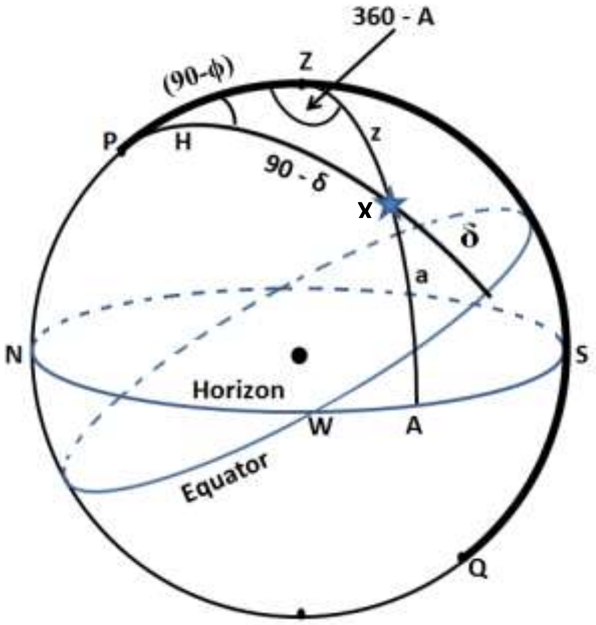
b- Hour angle (H):- is the angle measured from the observer's meridian westwards (for both north and south hemisphere observers) to the meridian through the star from 0^h to 24^h or from 0° to 360°

c- Right Ascension (α):- is the angular distance of the star from the vernal equinox to the meridian through the star. It is measured in hours, minutes or second.

$$\propto + H = S_t \ldots\ldots\ldots\ldots\ldots\ldots.1.47$$

Where S_t is the Sidereal time

Transformation of one coordinate system into another



A common problem in spherical astronomy is a wish to obtain star's coordinates in one system, given the coordinates in another system. The observer's latitude is usually known. In the spherical triangle *PZX*, we see that we require to find arc *PX* and angle *ZPX*. We calculate *PX* first of all, using the cosine formula because we know two sides *PZ, ZX* and the included angle *PZX*. Hence, we may write,

$$\cos PX = \cos PZ \cos ZX + \sin PZ \sin ZX \ cosPZX \dots\dots\dots .1.48$$

or $$\sin\delta = \sin\phi \sin a + \cos\phi \cos a \cos A \dots\dots\dots .1.49$$

This equation enables δ to be calculated.

A second application of the cosine formula gives

$$\cos ZX = \cos PZ \cos PX + \sin PZ \sin PX \cos ZPX \dots .1.50$$

Or $$\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos H \dots\dots\dots 1.51$$

Re-arranging, we obtain

$$\cos H = \frac{\sin a - \sin\phi \sin\delta}{\cos\phi \cos\delta} \dots\dots\dots\dots\dots .1.52$$

Giving H, since δ is now known.

Alternatively, using the four-parts formula with ZX, <PZX, PZ and <ZPX, we obtain

$$\cos PZ \cos PZX = \sin PZ \cot ZX - \sin PZX \cot ZPX \dots\dots\dots .1.53$$

Or $$\sin\phi \cos A = \cos\phi \tan a + \sin A \cot H \dots\dots\dots 1.54$$

Giving

$$\tan H = \frac{\sin A}{\sin\phi \cos A - \cos\phi \tan a} \dots\dots\dots\dots\dots\dots 1.55$$

Beginning	Sun position	Sun coordinate		constellation
		α	δ	
Spring 21st March	Vernal equinox	0^h	0°	Aries
Summer 22nd June	Summer solstic	6^h	23°27′	Cancer
Autumn 23rd September	Autumnal equinox	12^h	0°	Libra
Winter 22nd December	Winter solstice	18^h	-23°27′	Capricorn

Example:- A star of declination 42° 21′ N is observed when its hour angle is $8^h\ 16^m\ 42^s$. If the observer's latitude is 60° N, calculate the star's altitude and azimuth at the time of the observation.
We convert the hour angle into angular measure
$8^h\ 16^m\ 42^s$ =(8 x 15)° + (16/4)° + (40/4)′ + (2 x 15)″
=120° + 4° + 10′ + 30″
=124° 10.5′ = H

$$\cos H = \frac{\sin a - \sin \phi\ \sin \delta}{\cos \phi\ \cos \delta}$$

$$\sin a = \cos 30 \cos 47.65 - \sin 30 \sin 47.65 \cos 55.8253$$

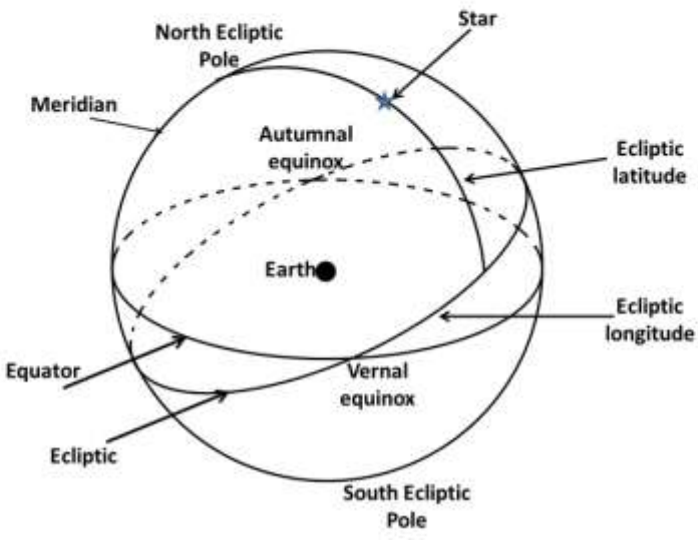
a= 22° 4.6′

$$\sin \delta = \sin \phi\ \sin a + \cos \phi\ \cos a\ \cos A$$

A= 41.2847°

3- Ecliptic system

This system is especially convenient in studying the movements of the planets in describing the Solar System. The two quantities specifying the position of an object on the celestial sphere in this system are ecliptic longitude and ecliptic latitude.



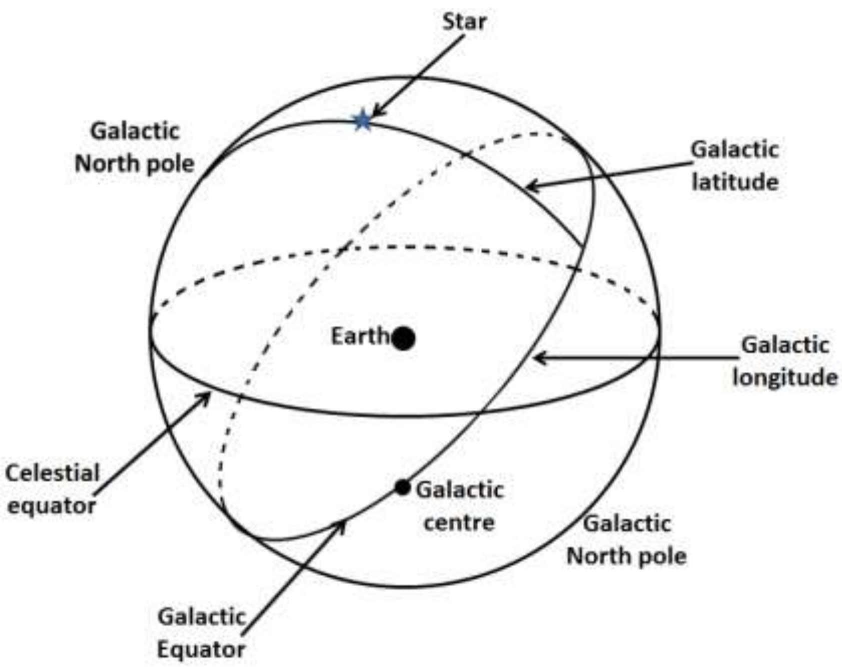
a- The ecliptic longitude: - is the angular distance between the Vernal equinox to the great circle through north ecliptic pole and the star.
b- The ecliptic latitude: - is the angular distance from the Ecliptic upward to the star.

This system is usually used to know the position of the Sun (which its latitude is zero), Moon and the planets.

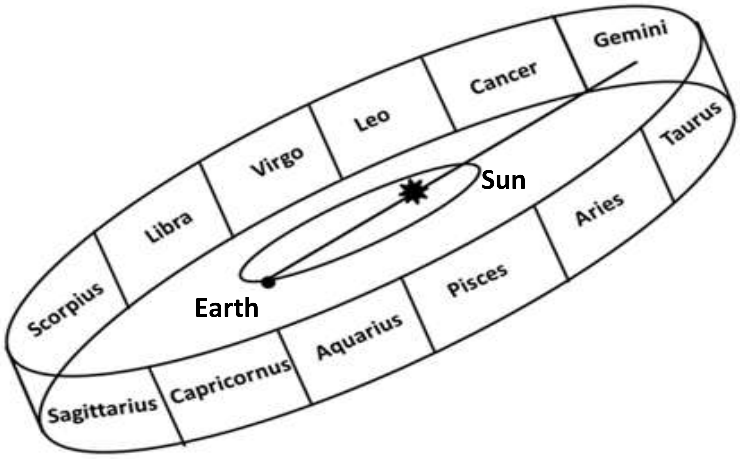
4- Galactic system

This system is used to study the objects in our Galaxy (Milky Way). The essential circle of this system is the Galactic equator and the coordinates are:

a- Galactic Longitude: - is the angular distance of an object eastward along the galactic equator from the galactic centre.
b- Galactic latitude: - is the angular distance of an object perpendicular to the galactic equator, positive to the north, negative to the south.



The zodiac:-is a circle of twelve 30° divisions of celestial longitude that are centred upon the ecliptic: the apparent path of the Sun across the celestial sphere over the course of the year. The paths of the Moon and visible planets also remain close to the ecliptic



The ecliptic:- is the apparent path of the Sun on the celestial sphere as seen from the Earth's centre, and also the plane of this path, which is essentially coplanar with the orbit of the Earth around the Sun. The path of the Sun is not normally noticeable from the Earth's surface because the Earth rotates, carrying the observer through the cycle of sunrise and sunset, obscuring the apparent position of the Sun against the background stars.

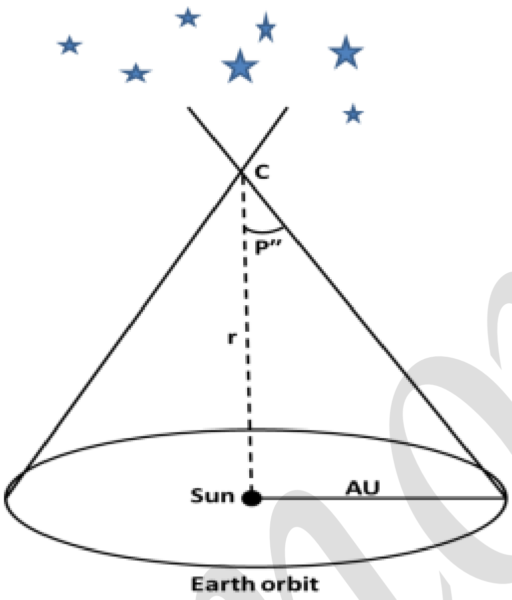
The astronomical units

Astronomical unit (AU):- is a unit of length equal to approximately 150 x 10^6 km or approximately the mean distance between the Sun and earth.

The light year: - A light-year is a unit of distance. It is the distance that light can travel in one year. Light moves at a velocity of about 300,000 (km/sec), so in one year, it can travel about 10 trillion km. More precisely, one light-year is equal to 9.45 x 10^{12} km.

The stellar parallax: - A nearby star's apparent movement against the background of more distant stars as the Earth revolves around the Sun is referred to as stellar parallax. The parallax is usually created by the different orbital positions of the Earth, which causes nearby stars to appear to move relative to more distant stars. By observing parallax, measuring angles and using geometry, one can determine the distance to various objects in space, typically stars, although other objects in space could be used. Because other stars are far away, the angle for measurement is small and the skinny triangle approximation can be applied, the distance to an object (measured in parsecs) is the reciprocal of the parallax (measured in arc-seconds):

$$d_{(pc)} = \frac{1}{p''}$$



The parsec: - is the more usual unit used by astronomers: it is the distance of a celestial body whose parallax is 1 arc second. It is usually used for the distant celestial objects.

1 parsec = 206265 AU

1 parsec = 30.86 x 10^{12} km approximately

1 parsec = 3.26 ly

1 ly = 9.45 x 10^{12} km